Bab I : Fungsi Transeden

* Fungsi Eksponen dan Logaritma

$^{b}\log x$

basis

$b > 0 \quad b \neq 1$

Logaritma merupakan kebalikan (invers) dari eksponen

$y = {}^{b}\log x$, maka diperoleh $b^{y} = x$

▶ Konsep :

$b^{y} = x$, kalo dalam logaritma kita ingin mencari :

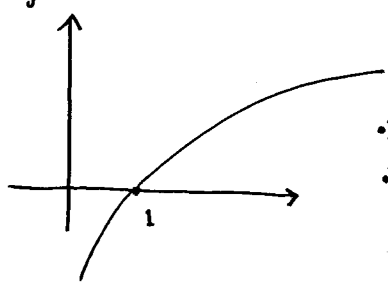
"b pangkat berapa hingga diperoleh x?"

Contoh : $2^{3} = 8 \Rightarrow {}^{2}\log 8 = 3$

▶ Kesamaan Logaritma :

$$^{b}\log b^{y} = y \cdot {}^{b}\log b$$
$$= y \cdot 1$$
$$= 1 \longrightarrow \boxed{^{b}\log b^{y} = 1}$$

▶ Grafik



$y = {}^{b}\log x$

-> Domain : $(0, +\infty)$

-> Range : $(-\infty, +\infty)$

▶ Sifat Umum Logaritma

i). $^{b}\log 1 = 0$

ii). $^{b}\log b = 1$

iii). $^{b}\log ac = {}^{b}\log a + {}^{b}\log c$

iv). $^{b}\log \frac{a}{c} = {}^{b}\log a - {}^{b}\log c$

v). $^{b}\log a^{n} = n \cdot {}^{b}\log a$

vi). $^{b}\log \frac{1}{c} = -{}^{b}\log c$

▶ Latihan :

① $^{3}\log 27 = \ldots$

② $\log \frac{x \cdot \sin^{2}x}{x+2} = \ldots$ (uraikan)

③ Ubah ke bentuk tunggal logaritma :

$\frac{1}{3}\log x - 2 \cdot \log(\sin 3x) + 2 = \ldots$?

* Logaritma Natural (ln)

$^{e}\log x \Longrightarrow \ln x$

salah — benar

▶ Sifat umum ln

$\ln 1 = 0 \;|\; \ln e = 1 \;|\; \ln e^{a} = a$

▶ Persamaan Umum

$y = \ln x$, maka sama saja $x = e^{y}$

▶ Sifat - Sifat ln

i). $\ln e^{x} = x$

ii). $e^{\ln x} = x$

iii). $\ln ac = \ln a + \ln c$

iv). $\ln \frac{1}{c} = -\ln c$ (ingat : $c^{-1} = \frac{1}{c}$)

v). $\ln \frac{a}{c} = \ln a - \ln c$

vi). $\ln a^{n} = n \ln a$

Latihan 1 :

① $\frac{1}{3}\ln x - \ln(x^{2} - 1) + 2\ln(x+3)$

②

* Turunan Fungsi Logaritma

▶ Persamaan Umum

$$\boxed{\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}} \quad \text{turunan}$$

$$\boxed{\int \frac{1}{u}\, du = \ln(u) + C} \quad \text{Integral}$$

Hati-hati!

$$\int \frac{1}{2x}\, dx = \ln(2x) + C$$

Cara penyelesaian :

misal : $u = 2x$

$du = 2\,dx \longrightarrow$

$$\int \frac{1}{u} \cdot \frac{1}{2}\, du$$
$$= \frac{1}{2}\int \frac{1}{u}\, du$$
$$= \frac{1}{2} \cdot \ln(u) + C$$

Latihan 2 :

ⓐ $\int \frac{x^{2}}{x^{3}-4}\, dx = \ldots$

ⓑ $\int \frac{x^{3}}{x^{2}+1}\, dx = \ldots$

Fungsi Eksponensial

$$b^x = \underbrace{b \cdot b \cdot b \cdot \ldots \cdot b}_{\text{Sebanyak } x \text{ kali}}$$

b = basis, x = eksponen

Sifat Operasi

a). $b^x \cdot b^y = b^{x+y}$

b). $\dfrac{b^x}{b^y} = b^{x-y}$

c). $b^0 = 1$

d). $b^{-x} = \dfrac{1}{b^x}$

e). $(b^x)^y = b^{xy}$

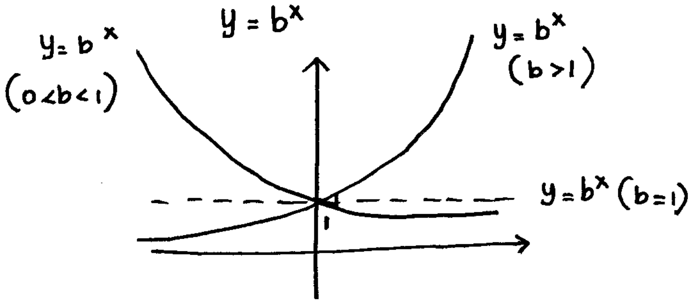
f). $(ab)^x = a^x b^x$

g). $\left(\dfrac{a}{b}\right)^x = \left(\dfrac{a^x}{b^x}\right)$

h). $b^{\frac{x}{y}} = \sqrt[y]{b^x}$

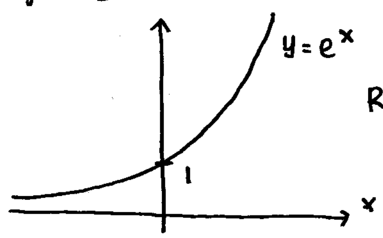
i). $\sqrt[y]{\sqrt[x]{b^k}} = b^{\frac{k}{x \cdot y}}$

Grafik Fungsi Eksponensial



Domain : $(-\infty, +\infty)$ Range: $(0, +\infty)$

Grafik $y = e^x$



Domain : $(-\infty, +\infty)$
Range : $(0, +\infty)$

Turunan Fungsi Eksponensial

$$\boxed{\frac{d}{dx}\left[b^u\right] = b^u \cdot \ln b \, \frac{du}{dx}}$$

Latihan :

(a) $\dfrac{d}{dx}\left[2^{x^2}\right] = \ldots$

Bentuk e^x

$$\boxed{\frac{d}{dx}\left[e^u\right] = e^u \cdot \frac{du}{dx}}$$

Latihan soal :

i). $\dfrac{d}{dx}\left[e^{-3x}\right] = \ldots$

ii). $\dfrac{d}{dx}\left[e^{\frac{x^2}{x+1}}\right] = \ldots$

iii). $\dfrac{d}{dx}\left[e^{-7x^2}\right] = \ldots$

iv). $\dfrac{d}{dx}\left[e^{\ln(x^3+1)}\right] = \ldots$

Integral Fungsi Eksponensial

$$\int b^u \, du = \frac{b^u}{\ln b} + C$$

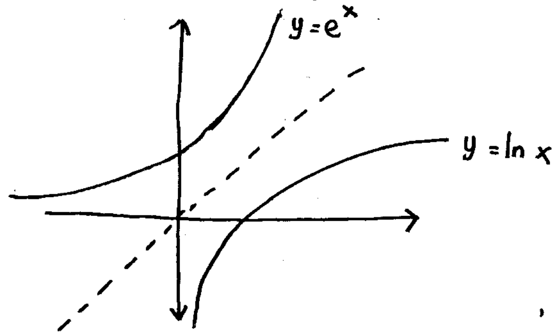
Latihan Soal :

a). $\int e^{-3x} \, dx = \ldots$

b). $\int x^2 \cdot e^{-2x^3} \, dx = \ldots$

Catatan !

e^x dan $\ln x$ merupakan fungsi saling invers.



Latihan soal!

1. Tentukan nilai x :

   a) $\dfrac{e^x}{1-e^x} = 2$ b). $e^{-2x} - 3e^{-x} = -2$

2. Dapatkan $\dfrac{dy}{dx}$ dari :

   a). $y = x\left[\ln(2x^2 - x)\right]^3$

   b). $y = \dfrac{x^2}{1+\ln x}$

   c). $y = (x^2 - 3x)^{\ln x}$

3. Hitunglah integral berikut

   a). $\int \dfrac{t+1}{t} \, dt$

   b). $\int \dfrac{dx}{\sqrt{x}(1-2\sqrt{x})}$

   c). $\int e^{3x}\sqrt{1+e^x} \, dx$ (ETS 2022/2023)

Soal - Soal ETS

1). $y = \dfrac{x^2\sqrt{6x+5}}{\sin(3x^2+1)\, e^x}$, $\dfrac{dy}{dx} = \ldots$

2). $y = \ln \dfrac{x^3 \sin 2x}{\sqrt{1+x^2}}$

a. Uraikan y dalam log natural

b. Dengan hasil a, dapatkan $\dfrac{dy}{dx}$

3). $\dfrac{dy}{dx}\left[x^3\left[\ln(3x^2+1)\right]^2\right] = \ldots$

4). $\int \dfrac{\sin 2\theta \cos 2\theta}{\cos^2 2\theta + 1} \, d\theta = \ldots$

$^3\log 27 = {}^3\log 3^3$

$= 3.\ {}^3\log 3$

$= 3$

(2) $\log \dfrac{x \sin^2 x}{x+2} = \log x + \log(\sin x)^2 - \log(x+2)$

$= \log x + 2 \log \sin x - \log(x+2)$

(3) $\frac{1}{3}\log x - 2\log(\sin 3x) + 2 = \log x^{1/3} - \log \sin^2 3x + \log 10^2$

$= \log \dfrac{\sqrt[3]{x}\ .\ 100}{\sin^2 3x}$

Logaritma Natural

(1) $\frac{1}{3}\ln x - \ln(x^2-1) + 2\ln(x+3)$

$= \ln x^{1/3} - \ln(x^2-1) + \ln(x+3)^2$

$= \ln \dfrac{\sqrt[3]{x}\ .\ (x+3)^2}{(x^2-1)}$

Latihan Turunan ln

(a) $\int \dfrac{x^2}{x^3-4}\,dx$

misal : $p = x^3 - 4$

$dp = 3x^2\,dx$

$\dfrac{dp}{3} = x^2\,dx$

$= \int \dfrac{dp}{3p}$

$= \frac{1}{3}.\ln p + C$

$= \frac{1}{3}\ln(x^3-4) + C$

(b) $\int \dfrac{x^3}{x^2+1}\,dx$

~~misal:~~

$x^3 \div (x^2+1) = x$

$x^3 + x$

$-x$

$x - \dfrac{x}{x^2+1}$

$\int \dfrac{x^3}{x^2+1}dx = \int x\,dx - \int \dfrac{x}{x^2+1}dx$

* $\int x\,dx = \frac{1}{2}x^2 + C$

* $\int \dfrac{x}{x^2+1}dx = \frac{1}{2}.\ln(x^2+1) + C$

misal : $x^2+1 = u$

$2x\,dx = du$

$x\,dx = \dfrac{du}{2}$

$\int \dfrac{du}{2u} = \frac{1}{2}\ln u$

$= \frac{1}{2}x^2 - \frac{1}{2}\ln(x^2+1) + C$

Latihan Turunan Eksponensial

(a) $\dfrac{d}{dx}[2^{x^2}] = 2^{x^2}.\ln 2.\ (2x)$

$= 2x.\ 2^{x^2}.\ln 2$

$= x.\ 2^{x^2+1}\ln 2$

Latihan Soal :

i). $\dfrac{d}{dx}[e^{-3x}] = e^{-3x}.\ -3$

$= -3e^{-3x}$

ii). $\dfrac{d}{dx}[e^{x^2/x+2}] = e^{\frac{x^2}{x+2}}.\ [2x(x+2) + x^2]$

$= [2x^2 + 4x + x^2]\, e^{x^2/x+2}$

$= [3x^2 + 4x]\, e^{\frac{x^2}{x+2}}$

iii) $\dfrac{d}{dx}[e^{-7x^2}] = e^{-7x^2}(-14x)$

$= -14x\,e^{-7x^2}$

iv). $\dfrac{d}{dx}[e^{\ln(x^3+1)}] = e^{\ln(x^3+1)}.\ \dfrac{1}{x^3+1}.\ 3x^2$

$= \dfrac{3x^2 e^{\ln(x^3+1)}}{x^3+1}$

* Integral Fungsi Eksponensial

a). $\int e^{-3x}dx =$

misal : $p = -3x$

$dp = -3dx$

$dx = -\dfrac{dp}{3}$

$\int e^p.\left(-\dfrac{dp}{3}\right)$

$= -\frac{1}{3}.e^p + C$

$= -\frac{1}{3}e^{-3x} + C$

b). $\int x^2.e^{-2x^3}dx$

misal : $u = -2x^3$

$du = -6x^2\,dx$

$\dfrac{-du}{6} = x^2\,dx$

$-\int \dfrac{e^u}{6}du$

$= -\frac{1}{6}e^u + C$

$= -\frac{1}{6}e^{-2x^3} + C$

Kumpulan Latihan Soal

1). Tentukan nilai x :

a). $\dfrac{e^x}{1-e^x} = 2 \longrightarrow e^x = 2 - 2e^x$

$3e^x = 2$

$e^x = \frac{2}{3}$

$x = \ln\left(\frac{2}{3}\right)$

b). $e^{-2x} - 3e^{-x} = -2$

misal : $e^{-x} = p$

$(e^{-x})^2 - 3(e^{-x}) + 2 = 0$

$p^2 - 3p + 2 = 0$

$(p-2)(p-1) = 0$

$p = 2 \rightarrow e^{-2}$

I.2 Fungsi Invers Trigonometri

Teorema. 1.2.2

> Jika f fungsi kontinu dan punya invers, maka f^{-1} juga kontinu.

Teorema 1.2.3

f punya invers dan nilai $f^{-1}(x)$ berubah-ubah pada selang tertentu dan f punya turunan tak nol seperti x berubah-ubah atas selang I, Maka f^{-1} dapat diturunkan pada I dan turunan f^{-1} diberikan oleh rumus:

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

* Contoh Penerapan Teorema:

fungsi $f(x) = x^5 + 7x^3 + 4x + 1$ punya invers. Tentukan turunan dari $f^{-1}(x)$.

a). Dengan persamaan $\frac{dy}{dx} = \frac{1}{dx/dy}$

Penyelesaian :

misal : $y = f^{-1}(x)$

$$x = f(y) = y^5 + 7y^3 + 4y + 1$$

$$\frac{dx}{dy} = 5y^4 + 21y^2 + 4$$

Diperoleh bahwa :

$$\frac{dy}{dx} = \frac{1}{dx/dy} = \frac{1}{5y^4 + 21y^2 + 4}$$

b). Dengan Turunan Implisit

$$\frac{d}{dx}[x] = \frac{d}{dx}[y^5 + 7y^3 + 4y + 1]$$

$$1 = 5y^4 \frac{dy}{dx} + 21y^2 \frac{dy}{dx} + 4\frac{dy}{dx}$$

$$1 = [5y^4 + 21y^2 + 4]\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{5y^4 + 21y^2 + 4}$$

Definisi Fungsi Invers

1. $\arcsin x \rightarrow \sin^{-1} x$

   Invers atau kebalikan fungsi $\sin x$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

2. $\arccos x \rightarrow \cos^{-1} x$

   $0 \leq x \leq \pi$

3. $\arctan \rightarrow \tan^{-1} x$

   $-\frac{\pi}{2} \leq x < \frac{\pi}{2}$

4. $\text{arcsec} \rightarrow \sec^{-1} x$

   $0 \leq x \leq \frac{\pi}{2}$ atau $\pi \leq x \leq \frac{3\pi}{2}$

Contoh Soal :

Dapatkan Inversnya!

a. $\sin^{-1}\left(\frac{1}{2}\right)$: $\sin y = \frac{1}{2}$, untuk $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

   maka diperoleh : $y = \frac{\pi}{6}$

b. $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

c. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

d. $\sin^{-1}(-1)$

▶ Rumus-Rumus Fungsi Invers Trigonometri

$\frac{d}{dx}[\sin^{-1} u] = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$	$\frac{d}{dx}[\cos^{-1} u] = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$
$\frac{d}{dx}[\tan^{-1} u] = \frac{1}{1+u^2} \frac{du}{dx}$	$\frac{d}{dx}[\cot^{-1} u] = -\frac{1}{1+u^2} \frac{du}{dx}$
$\frac{d}{dx}[\sec^{-1} u] = \frac{1}{u\sqrt{u^2-1}} \frac{du}{dx}$	$\frac{d}{dx}[\csc^{-1} u] = -\frac{1}{u\sqrt{u^2-1}}$

* Hafalkan bentuk kiri, yang kanan sama saja hanya perlu menambahkan negatif.

Latihan Soal!

Dapatkan $\frac{dy}{dx}$ dari beberapa soal berikut

a. $y = \sin^{-1}(x^3)$ b. $y = \sec^{-1}(e^x)$

c. $y = \sin^{-1}(P)$ d. $y = \cos^{-1}(2x+1)$

Contoh Pengerjaan!

$$\frac{dy}{dx}(\tan^{-1} x^2) = \frac{1}{1+x^4} \cdot (2x)$$

$$= \frac{2x}{1+x^4}$$

//

Rumus Integrasi Fungsi Invers Trigonometri

* karena integral adalah kebalikan turunan, cukup hafalin salah satu turunan atau integral.

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$

Contoh Soal !

① $\int \frac{e^x}{4+e^{2x}}\,dx$

misal : $p = e^x$

$dp = e^x\,dx$

$dx = \frac{dp}{e^x}$

$$\int \frac{e^x}{2^2+(e^x)^2}\,dx = \int \frac{\cancel{e^x}}{2^2+p^2}\cdot\frac{dp}{\cancel{e^x}} = \frac{1}{2}\tan^{-1}\frac{p}{2} + C = \frac{1}{2}\tan^{-1}\frac{e^x}{2} + C$$

② $\int \frac{\sin\theta}{\cos^2\theta+1}\,d\theta$

Misal : $p = \cos\theta$

$dp = -\sin\theta\,d\theta$

$-dp = \sin\theta\,d\theta$

$$\int \frac{-dp}{p^2+1} = -\tan^{-1}p + C = -\tan^{-1}(\cos\theta) + C$$

③ $\int \frac{dx}{\sqrt{1-4x^2}}$

misal : $p = 2x$

$dp = 2\,dx$

$dx = \frac{dp}{2}$

$$\int \frac{dp}{2\sqrt{1-p^2}} = \frac{1}{2}\int\frac{dp}{\sqrt{1-p^2}} = \frac{1}{2}\sin^{-1}p + C = \frac{1}{2}\sin^{-1}(2x) + C$$

Trik Cara menjawab :

1). Tebak kemungkinan bentuk integral

2). Manipulasi bentuk hingga sesuai persamaan

- Pemisalan

3). Substitusi pemisalan ke soal

Latihan soal !

a). $\int \frac{dx}{1+16x^2} = \ldots$

b). $\int \frac{t}{t^4+1}\,dt$

c). $\int \frac{\sec^2 x\,dx}{\sqrt{1-\tan^2 x}}$

d). $\int_1^2 \frac{1}{\sqrt{x}\sqrt{4-x}}\,dx$

I.3 Fungsi Hiperbolik

Definisi 1.3.1

$$\sinh x = \frac{e^x-e^{-x}}{2} \quad \text{dan} \quad \cosh x = \frac{e^x+e^{-x}}{2}$$

Berdasarkan Definisi 1.3.1 tersebut, diperoleh fungsi hiperbolik lainnya :

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x-e^{-x}}{e^x+e^x} \qquad \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x+e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x+e^{-x}}{e^x-e^{-x}} \qquad \operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x-e^{-x}}$$

Kesamaan Hiperbolik

$\cosh^2 x - \sinh^2 x = 1$

$\cosh(-x) = \cosh(x)$

$\sinh(-x) = -\sinh(x)$

$\sinh 2x = 2\sinh x\cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh 2x = 2\sinh^2 x + 1$

$= 2\cosh^2 x - 1$

$\sinh(x\pm y) = \sinh x\cosh y \pm \cosh x\sinh y$

$\cosh(x\pm y) = \cosh x\cosh y \pm \sinh x\sinh y$

Rumus Turunan Hiperbolik

$\frac{d}{dx}[\sinh u] = \cosh u\,\frac{du}{dx}$

$\frac{d}{dx}[\cosh u] = \sinh u\,.\frac{du}{dx}$

$\frac{d}{dx}[\tanh u] = \operatorname{sech}^2 u\,\frac{du}{dx}$

$\frac{d}{dx}[\coth u] = -\operatorname{csch}^2 u\,\frac{du}{dx}$

$\frac{d}{dx}[\operatorname{sech} u] = -\operatorname{sech} u\tanh u\,\frac{du}{dx}$

$\frac{d}{dx}[\operatorname{csch} u] = -\operatorname{csch} u\coth u\,\frac{du}{dx}$

Contoh Soal !

Dapatkan $\frac{dy}{dx}$

(a). $\frac{d}{dx}[\sinh(4x-8)] = \cosh(4x-8).\,4$

$= 4\cosh(4x-8)$

(b). $\frac{d}{dx}[\sinh(\cos 3x)] = \cosh(\cos 3x).\,(-3\sin 3x)$

$= -3\sin 3x\,[\cosh(\cos 3x)]$

↳ Penjabaran :

$u = \cos 3x$

$du = -3\sin 3x\,dx$

$\frac{du}{dx} = -3\sin 3x$

$= \cosh(\cos 3x).\,[-3\sin 3x]$

$= -3\sin 3x\,[\cosh(\cos 3x)]$

Rumus Integral :

$$\int \sinh u\, du = \cosh u + C \qquad \int \operatorname{csch}^2 u\, du = -\coth u + C$$

$$\int \cosh u\, du = \sinh u + C \qquad \int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u + C$$

$$\int \operatorname{sech}^2 u\, du = \tanh u + C \qquad \int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u + C$$

Contoh soal :

$$\int \sinh^5 x \cosh x\, dx = \int p^5 . dp$$

misal : $p = \sinh x$

$dp = \cosh x\, dx$

$$= \frac{1}{6} p^6 + C$$

$$= \frac{1}{6} \sinh^6 x + C$$

Latihan Soal !

1). Dapatkan $\frac{dy}{dx}$:

a). $y = \coth(\ln x)$ c). $y = \sinh^3(2x)$

b). $y = \sinh(2x)$ d). $y = \sinh(\cos 3x)$

2). Hitunglah Integral dari :

a). $\int \operatorname{csch}^2(3x)\, dx$ c). $\int \coth^2 x \operatorname{csch}^2 x\, dx$

b). $\int \sqrt{\tanh x} \operatorname{sech}^2 x\, dx$ d). $\int \frac{\sinh 2x}{3 + 5\cosh 2x}\, dx$

▶ Bentuk Logaritma Fungsi Invers Hiperbolik

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right) \qquad \cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$$

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x} \qquad \coth^{-1} x = \frac{1}{2} \ln \frac{x+1}{x-1}$$

$$\operatorname{sech}^{-1} x = \ln\left[\frac{1 + \sqrt{1-x^2}}{x}\right] \qquad \operatorname{csch}^{-1} x = \ln\left[\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right]$$

▶ Turunan Fungsi Invers Hiperbolik

$$\frac{d}{dx}[\sinh^{-1} x] = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}}, (x>1)$$

$$\frac{d}{dx}[\tanh^{-1} x] = \frac{1}{1-x^2}, (|x|<1) \qquad \frac{d}{dx}[\coth^{-1} x] = \frac{1}{1-x^2}, (|x|>1)$$

$$\frac{d}{dx}[\operatorname{sech}^{-1} x] = -\frac{1}{x\sqrt{1-x^2}} (0<x<1) \qquad \frac{d}{dx}[\operatorname{csch}^{-1} x] = -\frac{1}{|x|\sqrt{1+x^2}}, x \neq 0$$

▶ Integral Fungsi Invers Hiperbolik

$$\int \frac{du}{\sqrt{1+u^2}} = \sinh^{-1} u + C \qquad \int \frac{du}{\sqrt{u^2-1}} = \cosh^{-1} u + C \quad (u>1)$$

$$\int \frac{du}{1-u^2} = \begin{cases} \tanh^{-1} u + C, & \text{jika } |u|<1 \\ \coth^{-1} u + C, & \text{jika } |u|>1 \end{cases}$$

$$\int \frac{du}{u\sqrt{1-u^2}} = -\operatorname{sech}^{-1} |u| + C$$

$$\int \frac{du}{u\sqrt{1+u^2}} = -\operatorname{csch}^{-1} |u| + C$$

Contoh Soal !

Dapatkan $\frac{dy}{dx}$ dari persamaan berikut.

① $y = \sinh^{-1}\left(\frac{1}{3}x\right)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+\left(\frac{1}{3}x\right)^2}} \cdot \frac{1}{3}$$

$$= \frac{1}{3} \cdot \frac{1}{\sqrt{1+\left(\frac{1}{3}x\right)^2}} \quad //$$

② $y = \tanh^{-1}(x^2)$

misal $\rightarrow u = x^2$ $\rightarrow$ akibatnya :

$du = 2x\, dx$ $\qquad y = \tanh^{-1} u$

$\frac{du}{dx} = 2x$ $\qquad \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Jawab : $\frac{dy}{dx} = \frac{1}{1-u^2} \cdot 2x$

$$= \frac{2x}{1-x^4} \quad //$$

Latihan Soal !

a). $y = \cosh^{-1}(2x+1)$ d). $y = \cosh^{-1}(\cosh x)$

b). $y = \coth^{-1}(\sqrt{x})$ e). $y = e^x \operatorname{sech}^{-1} x$

c). $y = \operatorname{csch}^{-1}(e^x)$ f). $y = x^2 (\sinh^{-1} x)^3$

Contoh Soal!

$$\int \frac{dx}{\sqrt{1+9x^2}}$$

misal : $3x = u$

$3\,dx = du$

$dx = \frac{du}{3}$

$$\frac{1}{3} \int \frac{du}{\sqrt{1+u^2}}$$

$$= \frac{1}{3} \sinh^{-1} u + C$$

$$= \frac{1}{3} \sinh^{-1} (3x) + C$$

$$\int \frac{\sin\theta \, d\theta}{\sqrt{1+\cos^2\theta}}$$

misal : $p = \cos\theta$

$dp = -\sin\theta\, d\theta$

$-dp = \sin\theta\, d\theta$

$$\int - \frac{dp}{\sqrt{1+p^2}}$$

$$= -\sinh^{-1} p + C$$

$$= -\sinh^{-1} (\cos\theta) + C$$

Latihan Soal!

(a). $\displaystyle\int \frac{dx}{x\sqrt{1+x^6}}$ (b). $\displaystyle\int \frac{dx}{\sqrt{1-e^{2x}}}$

Kumpulan Soal-Soal

Kuis, ETS, EAS

1). Hitunglah Integral dari :

$$\int \frac{\sinh 2x}{3 + 5\cosh 2x}\, dx$$ (ETS 2023)

2). Hitunglah Integral dari :

$$\int \sinh^6 3t \cosh 3t \, dt$$ (ETS 2023)